

The Strong Decay Patterns of the 1^{-+} Exotic Hybrid Mesons

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We calculate the coupling constants of the decay modes $1^{-+} \rightarrow \rho\pi, f_1\pi, b_1\pi, \eta\pi, \eta'\pi, a_1\pi, f_1\eta$ within the framework of the light-cone QCD sum rule. Then we calculate the partial width of these decay channels, which differ greatly from the existing calculations using phenomenological models. For the isovector 1^{-+} state, the dominant decay modes are $\rho\pi, f_1\pi$. For its isoscalar partner, its dominant decay mode is $a_1\pi$. We also discuss the possible search of the 1^{-+} state at BESIII, for example through the decay chains $J/\psi(\psi') \rightarrow \pi_1 + \gamma$ or $J/\psi(\psi') \rightarrow \pi_1 + \rho$ where π_1 can be reconstructed through the decay modes $\pi_1 \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$ or $\pi_1 \rightarrow f_1(1285)\pi^0$. Hopefully the present work will be helpful to the experimental establishment of the 1^{-+} hybrid meson.

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I. INTRODUCTION

Quark model has been proved to be very successful in the classification of hadrons and calculation of hadron spectrum and their other properties. Yet Quantum Chromodynamics (QCD), which is widely accepted as the fundamental theory of the strong interaction, does not prohibit the existence of those hadron states which can not be accommodated in the conventional quark model. These non-conventional hadrons include multi-quark states ($qq\bar{q}\bar{q}, qq\bar{q}\bar{q}, \dots$), glueballs (gg, ggg, \dots), and hybrids ($q\bar{q}g$). Searching for non-conventional hadrons experimentally and studies on their properties have attracted much interests over the past few decades. With these efforts one aims to explore the nonperturbative aspects of QCD in the low energy sector.

Some non-conventional hadrons are totally “exotic”, namely their J^{PC} quantum numbers are excluded by the conventional quark model. For example, a conventional $q\bar{q}$ meson possesses parity $P = (-1)^{L+1}$ and C -parity $C = (-1)^{L+S}$, where L and S are the orbital angular momentum and spin of its componential quark and antiquark, respectively. Apparently, $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. are impossible for conventional mesons. Hadrons with these J^{PC} quantum numbers are exotic states, which are widely studied since they do not mix with conventional hadrons.

Recently, COMPASS collaboration observed a resonance with exotic quantum numbers $J^{PC} = 1^{-+}$ at $(1660 \pm 10_{-64}^{+0})$ MeV/ c^2 with a width of $(269 \pm 21_{-64}^{+42})$ MeV/ c^2 [1]. In literature, three isovector $J^{PC} = 1^{-+}$ exotic mesons, namely $\pi_1(1400)$, $\pi_1(1600)$, and $\pi_1(2015)$, have been reported. Several groups observed $\pi_1(1400)$ in the $\eta\pi^-$ system [2]. $\pi_1(1600)$ was first reported through a combined study of the $\eta'\pi^-$, $f_1\pi^-$, and $\rho^0\pi^-$ channels by VES in Ref. [3] and later in the $b_1\pi$ final state in Ref. [4]. E852 collaboration observed $\pi_1(1600)$ by carrying out the partial wave analysis in the $\pi^+\pi^-\pi^-$ final state [5] and examining the $\eta'\pi^-$ final state in the reaction $\pi^-p \rightarrow p\eta'\pi^-$ [6]. They also found it in the final states $f_1\pi$ [7] and $b_1\pi$ [8]. The Crystal Barrel collaboration analyzed the reaction $\bar{p}p \rightarrow \omega\pi^+\pi^-\pi^0$ and observed the $\pi_1(1600)$ in the $b_1\pi$ channel in Ref. [9]. E852 also reported another 1^{-+} meson $\pi_1(2015)$ [7, 8].

The 1^{-+} hybrids have been studied in a few different theoretical schemes. The mass of the lowest-lying 1^{-+} hybrid meson was predicted to be around 1.9 GeV in the flux tube model [17]. Llanes-Estrada and Cotanch predicted the hybrid mass to be above 2.0 GeV utilizing a QCD inspired Coulomb gauge Hamiltonian [10]. H.-C. Kim and Y. Kim investigated the 1^{-+} hybrid within the framework of an AdS/QCD model [11]. Early studies with the leading order QCD sum rule estimated the mass of the 1^{-+} hybrid to be $(1.6 \sim 2.1)$ GeV [21]. The inclusion of the radiative corrections changed this estimate to 1.26 ± 0.15 GeV [22], $(1.6 \sim 1.7)$ GeV [23], and $1.81(6)$ GeV [24]. The Lattice QCD prediction for the mass of 1^{-+} falls into a wide range of $(1.5 \sim 2.2)$ GeV [14].

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The 1^{-+} hybrids were shown to exist as narrow resonant states in the large N_c limit of QCD [12]. The coupling of a neutral hybrid $\{1, 3, 5 \dots\}^{-+}$ to two neutral (hybrid) mesons with the same J^{PC} and $J = 0$ were studied in the large N_c limit of QCD in Ref. [13]. Cook and Fiebig presented a decay width calculation on lattice for the channel $1^{-+} \rightarrow a_1 \pi$ in Ref. [15]. The partial widths of the ground 1^{-+} hybrid to $b_1 \pi$ and $f_1 \pi$ were predicted to be 400(120) MeV and 90(60) MeV, respectively in Ref. [16]. The strong decay properties were explored in Ref. [18] (IKP) within the same framework. Page, Swanson, and Szczepaniak [19] (PSS) extended the original IKP flux tube model and studied the strong decays of hybrid mesons with different J^{PC} quantum numbers, including those of the 1^{-+} hybrids. Burns and Close [20] compared the flux tube model and Lattice QCD for the S -wave decay of the 1^{-+} hybrid and found excellent agreement.

Some decay modes of the isoscalar and isovector 1^{-+} hybrids were studied using the three-point function sum rule [25]. In Ref. [26], the decay widths of the 1^{-+} hybrid were calculated using a three-point function sum rule evaluated at the symmetric Euclidean point. The mass of the strange hybrid was also studied in this paper within a light quark expansion formalism. The partial width of the channel $1^{-+} \rightarrow \rho \pi$ was predicted to be rather broad by using the three-point function at the symmetric point [26, 27]. Zhu reexamined the decay channels $1^{-+} \rightarrow \rho \pi, f_1 \pi$ by using the light-cone QCD sum rules and reduced the partial width of $1^{-+} \rightarrow \rho \pi$ significantly [28].

In this work, we employ Light-cone QCD sum rule (LCQSR) [29] to calculate the various coupling constants of the decay modes $1^{-+} \rightarrow \rho \pi, f_1 \pi, b_1 \pi, \eta \pi, \eta' \pi, a_1 \pi, f_1 \eta$. The LCQSR is different from the traditional Shifman-Vainshtein-Zakharov (SVZ) sum rule [30], where the short-distance OPE expansion is made. The OPE expansion of the LCQSR is performed near the light cone. With the extracted coupling constants, we calculate the partial widths of these modes and compare them with the existing results obtained using other approaches. We also suggest the possible search for the 1^{-+} hybrid mesons at BESIII.

The paper is organized as follows. We illustrate the formalism and derive the LCQSR for the $\pi_1 \rightarrow \rho \pi$ coupling constant in Sec. II. We present the sum rules for the $\pi_1 \rightarrow f_1 \pi, b_1 \pi$ and $\pi_1 \rightarrow \eta \pi, \eta' \pi$ coupling constants in Sec. III and IV respectively. The decay widths of the isovector and isoscalar 1^{-+} states are presented in Sec. V and VI respectively. We discuss the possible search of these exotic states at BESIII in Sec. VII. The last section is a short summary. The light cone distribution amplitudes of the pion which are employed in the present calculation are collected in the appendix. Readers who are not interested in the technical details may skip Secs. II-IV and go to last three sections directly.

II. SUM RULES FOR THE $\pi_1 \rightarrow \rho \pi$ COUPLING CONSTANT

We denote the isovector and the isoscalar $J^{PC} = 1^{-+}$ hybrid meson by π_1 and $\tilde{\pi}_1$, respectively. The adopted interpolating current for π_1 reads

$$J_{\mu}^{\pi_1}(x) = \frac{1}{\sqrt{2}} \left[\bar{u}(x) \frac{\lambda^a}{2} g_s G_{\mu\nu}^a(x) \gamma^{\nu} u(x) - \bar{d}(x) \frac{\lambda^a}{2} g_s G_{\mu\nu}^a(x) \gamma^{\nu} d(x) \right]. \quad (1)$$

The overlapping amplitude \tilde{f}_{π_1} between the above interpolating current and π_1 is defined as

$$\langle 0 | J_{\mu}^{\pi_1}(0) | \pi_1(p, \lambda) \rangle = \tilde{f}_{\pi_1} \eta_{\mu}^{\lambda}, \quad (2)$$

where η_{μ} is the polarization vector of π_1 .

We consider the following correlation functions in our calculation:

$$\Pi_{\rho}(k^2, p^2) = i \int d^4 x e^{ik \cdot x} \langle \pi(q) | T \{ J_{\alpha}^{\rho}(x) J_{\beta}^{\pi_1 \dagger}(0) \} | 0 \rangle = i \varepsilon_{\alpha\beta\gamma\delta} k^{\gamma} q^{\delta} G_{\rho}(k^2, p^2) + \dots, \quad (3)$$

where $p = k + q$. The interpolating current for the ρ meson is $J_{\mu}^{\rho}(x) = \bar{d}(x) \gamma_{\mu} u(x)$ and we have $\langle 0 | J_{\mu}^{\rho}(0) | \rho(k, \lambda) \rangle = f_{\rho} m_{\rho} \epsilon_{\mu}^{\lambda}$ where ϵ_{μ} is the polarization vector of the ρ meson. The decay amplitude of the channel $\pi_1 \rightarrow \rho \pi$ can be written as

$$\mathcal{M}(\pi_1 \rightarrow \rho \pi) = \varepsilon_{\alpha\beta\gamma\delta} \epsilon^{*\alpha} \eta^{\beta} k^{\gamma} q^{\delta} g_{\rho}, \quad (4)$$

where k and q are the momentum of the final ρ and π , respectively. When $k^2, p^2 \ll 0$, $G_{\rho}(k^2, p^2)$ can be calculated by operator product expansion(OPE) near the light-cone $x^2 = 0$, with the π light-cone wave functions as its input. Furthermore, $G_{\rho}(k^2, p^2)$ can be related to g_{ρ} by the dispersion relation

$$G_{\rho}(k^2, p^2) = \int_0^{\infty} ds_1 \int_0^{\infty} ds_2 \frac{\rho(s_1, s_2)}{(s_1 - k^2 - i\epsilon)(s_2 - p^2 - i\epsilon)} + \int_0^{\infty} ds_1 \frac{\rho_1(s_1)}{s_1 - k^2 - i\epsilon} + \int_0^{\infty} ds_2 \frac{\rho_2(s_2)}{s_2 - p^2 - i\epsilon} + \dots, \quad (5)$$

where

$$\rho(s_1, s_2) = f_\rho \tilde{f}_{\pi_1} m_\rho g_\rho \delta(s_1 - m_\rho^2) \delta(s_2 - m_{\pi_1}^2) + \dots \quad (6)$$

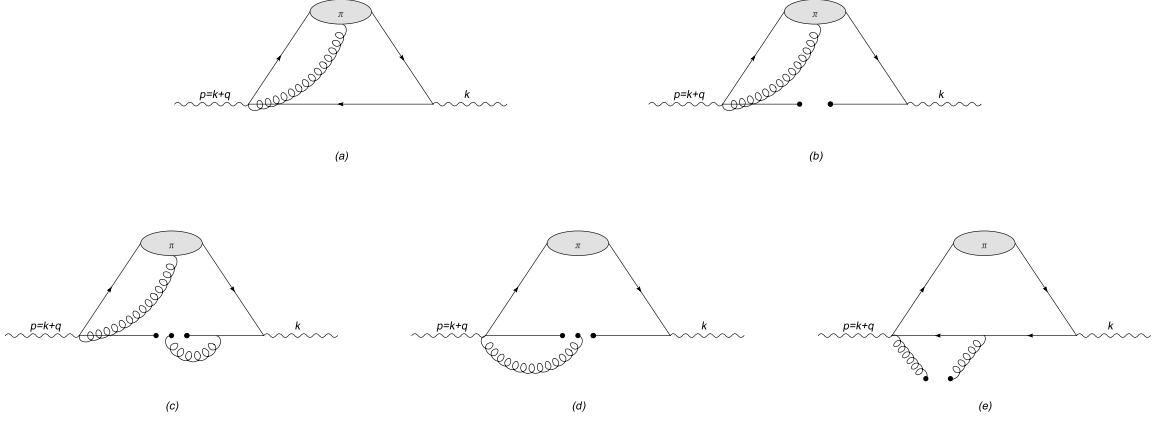


FIG. 1: The Feynman diagrams for $\Pi_\rho(k^2, p^2)$.

After invoking the double Borel transformation $\mathcal{B}_{k^2}^{M_1^2} \mathcal{B}_{p^2}^{M_2^2}$, we extract the double dispersion relation part of Eq. (5):

$$\begin{aligned} & f_\rho \tilde{f}_{\pi_1} m_\rho g_\rho e^{\bar{u}_0 m_\rho^2/M^2 + u_0 m_{\pi_1}^2/M^2} + \dots \\ &= e^{u_0 \bar{u}_0 m_\pi^2/M^2} \left\{ \frac{f_\pi}{\sqrt{2}} (\mathcal{A}_\perp^{[\alpha_2]} - \mathcal{A}_\perp^{[\alpha_1]} - \mathcal{V}_\parallel^{[\alpha_1]} - \mathcal{V}_\parallel^{[\alpha_2]} + \mathcal{V}_\perp^{[\alpha_1]} + \mathcal{V}_\perp^{[\alpha_2]}) m_\pi^2 M^2 - \frac{f_\pi}{36\sqrt{2}} [\phi_\pi(u_0) + \phi_\pi(\bar{u}_0)] \langle g_s^2 G^2 \rangle \right\} \\ &\approx \frac{f_\pi}{\sqrt{2}} (\mathcal{A}_\perp^{[\alpha_2]} - \mathcal{A}_\perp^{[\alpha_1]} - \mathcal{V}_\parallel^{[\alpha_1]} - \mathcal{V}_\parallel^{[\alpha_2]} + \mathcal{V}_\perp^{[\alpha_1]} + \mathcal{V}_\perp^{[\alpha_2]}) m_\pi^2 M^2 - \frac{f_\pi}{36\sqrt{2}} [\phi_\pi(u_0) + \phi_\pi(\bar{u}_0)] \langle g_s^2 G^2 \rangle, \end{aligned} \quad (7)$$

where

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2}, \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}, \quad (8)$$

and $\bar{x} \equiv 1 - x$. Hereafter we ignore the factor $e^{u_0 \bar{u}_0 m_\pi^2/M^2}$ because $m_\pi^2/M^2 < 0.01$ in our calculations. The definitions of $\mathcal{F}^{[\alpha_i]}$ s are

$$\begin{aligned} \mathcal{F}^{[\alpha_1]} &\equiv \int_0^{\bar{u}_0} \mathcal{F}(\alpha_1, u_0, \bar{u}_0 - \alpha_1) d\alpha_1, \\ \mathcal{F}^{[\alpha_2]} &\equiv \int_0^{\bar{u}_0} \mathcal{F}(u_0, \alpha_2, \bar{u}_0 - \alpha_2) d\alpha_2, \\ \mathcal{F}^{[\alpha_1, u]} &\equiv \int_0^{\bar{u}_0} \int_0^u \mathcal{F}(\alpha_1, \alpha'_2, \bar{\alpha}_1 - \alpha'_2) d\alpha'_2 d\alpha_1, \\ \mathcal{F}^{[\alpha_2, u]} &\equiv \int_0^{\bar{u}_0} \int_0^u \mathcal{F}(\alpha'_1, \alpha_2, \bar{\alpha}_2 - \alpha'_1) d\alpha'_1 d\alpha_2. \end{aligned} \quad (9)$$

Here the Borel transformation is defined as

$$\mathcal{B}_{k^2}^{M_1^2} [f(k^2)] = \lim_{n \rightarrow \infty} \frac{(-k^2)^{n+1}}{n!} \left(\frac{d}{dk^2} \right)^n f(k^2) \Big|_{k^2 = -nM_1^2}. \quad (10)$$

The quark propagator used in the OPE of $G_\rho(k^2, p^2)$ is

$$iS(x) = \frac{i\not{x}}{2\pi^2 x^4} + \frac{i}{32\pi^2} \frac{\lambda^n}{2} g_s G_{\mu\nu}^n \frac{1}{x^2} (\sigma^{\mu\nu} \not{x} + \not{x} \sigma^{\mu\nu}) - \frac{\langle \bar{q}q \rangle}{12} - \frac{m_0^2 \langle \bar{q}q \rangle x^2}{192} + \dots$$

We present the Feynman diagrams corresponding to the quark-level calculation of $\Pi_\rho(k^2, p^2)$ in Fig. 1.

The spectral density $\rho(s_1, s_2)$ can be derived from the dispersion relation Eq. (5) after two continuous double Borel transformation of $G_\rho(k^2, p^2)$:

$$\rho(s_1, s_2) = \mathcal{B}_{\sigma_1}^{\frac{1}{s_1}} \mathcal{B}_{\sigma_2}^{\frac{1}{s_2}} \mathcal{B}_{k^2}^{\frac{1}{s_1}} \mathcal{B}_{p^2}^{\frac{1}{s_2}} G_\rho(k^2, p^2). \quad (11)$$

According to quark-hadron duality, we can subtract the contribution of the excited states and the continuum from Eq. (7) and arrive at

$$f_\rho \tilde{f}_{\pi_1} m_\rho g_\rho e^{\bar{u}_0 m_\rho^2/M^2 + u_0 m_{\pi_1}^2/M^2} = \int_0^{s_{01}} ds_1 \int_0^{s_{02}} ds_2 e^{-s_1 \sigma_1} e^{-s_2 \sigma_2} \mathcal{B}_{\sigma_1}^{\frac{1}{s_1}} \mathcal{B}_{\sigma_2}^{\frac{1}{s_2}} \mathcal{B}_{k^2}^{\frac{1}{s_1}} \mathcal{B}_{p^2}^{\frac{1}{s_2}} G_\rho(k^2, p^2), \quad (12)$$

where s_{01} and s_{02} are the continuum thresholds of the mass rules of the ρ meson and the hybrid state π_1 , respectively.

The large mass difference between the π_1 hybrid and the ρ meson inclines us to work at an asymmetric point of the Borel parameter M_1^2 and M_2^2 , leading to a sophisticated subtraction of the continuum contribution [34]. The terms of $\mathcal{B}_{k^2}^{\frac{1}{s_1}} \mathcal{B}_{p^2}^{\frac{1}{s_2}} G_\rho(k^2, p^2)$ have general form $cu_0^m (M^2)^n = c\sigma_2^m / (\sigma_1 + \sigma_2)^{m+n}$. Here we assume $m, n > 0$ to illustrate the procedure of the continuum subtraction.

$$\begin{aligned} & \int_0^{s_{01}} ds_1 \int_0^{s_{02}} ds_2 e^{-s_1 \sigma_1} e^{-s_2 \sigma_2} \mathcal{B}_{\sigma_1}^{\frac{1}{s_1}} \mathcal{B}_{\sigma_2}^{\frac{1}{s_2}} \frac{\sigma_2^m}{(\sigma_1 + \sigma_2)^{m+n}} \\ &= \int_0^{s_{01}} ds_1 \int_0^{s_{02}} ds_2 e^{-s_1 \sigma_1} e^{-s_2 \sigma_2} \frac{1}{\Gamma(m+n)} \left[-\frac{\partial \delta(s_1 - s_2)}{\partial s_1} \right]^m s_1^{m+n-1} \\ &= 2 \int_0^{s_{01}} ds_+ \int_{-s_+}^{s_+} ds_- e^{-s_+ M_-^2} e^{s_- M_-^2} \frac{(s_+ - s_-)^{m+n-1}}{2^m \Gamma(m+n)} \left(\frac{\partial}{\partial s_-} \right)^m \delta(2s_-) \\ &= \frac{M_-^{2n}}{2^m} \sum_{i=0}^m \frac{m!}{i!(m-i)!} (2u_0 - 1)^i f_{n-1+i} \left(\frac{s_{01}}{M_-^2} \right), \end{aligned} \quad (13)$$

where $s_+ = (s_1 + s_2)/2$, $s_- = (s_2 - s_1)/2$, $1/M_-^2 = 1/M_1^2 - 1/M_2^2$ and we assume $s_{01} < s_{02}$. $f_n(x)$ is the subtraction function defined as

$$f_n(x) = 1 - e^{-x} \sum_{i=0}^n \frac{x^i}{i!}. \quad (14)$$

The 1^{--} hybrid has not been firmly established experimentally. Both theoretical predictions and experimental measurements suggest that the mass of π_1 falls within the range $1.6 \sim 2.0$ GeV. In this work the mass of π_1 is taken to be $m_{\pi_1} = 1.6, 1.8$ and 2.0 GeV. We adopt $\tilde{f}_{\pi_1} = 0.15$ GeV⁴ in our numerical analysis [23]. The π decay constant $f_\pi = 131$ MeV. The mass and the decay constant of the ρ meson are $m_\rho = 0.77$ GeV and $f_\rho = 0.216$ GeV. $\mu_\pi \equiv m_\pi^2/(m_u + m_d) = (1.573 \pm 0.174)$ GeV is given in Ref. [31].

The parameters appear in the π distribution amplitudes are listed below [31]. We use the values at the scale $\mu = 1$ GeV in our calculation.

a_2	η_3	ω_3	η_4	ω_4	h_{00}	v_{00}	a_{10}	v_{10}	h_{01}	h_{10}
0.25	0.015	-1.5	10	0.2	-3.33	-3.33	5.14	5.25	3.46	7.03

It is reasonable to let $M_1^2 = \beta m_\rho$ and $M_2^2 = \beta m_{\pi_1}$, where β is a dimensionless scale parameter. Then we have $u_0 = m_\rho^2/(m_\rho^2 + m_{\pi_1}^2)$ and $M^2 = \beta m_\rho^2 m_{\pi_1}^2 / (m_\rho^2 + m_{\pi_1}^2)$.

From the requirement of the convergence of the OPE and the requirement that the pole contribution is larger than 40%, we get the working interval of the Borel parameter M^2 . The resulting sum rule is plotted with $s_{01} = 1.5, 1.7, 1.9$ GeV² in Fig. 2 in the case of $m_{\pi_1} = 1.6$ GeV. The sum rules for $m_{\pi_1} = 1.8, 2.0$ GeV are similar. The numerical values of g_ρ are presented here with their variations determined by the working interval of the Borel parameter $2.3 < M^2 < 2.7$ GeV² and the range of the threshold $1.5 < s_{01} < 1.9$ GeV².

$m_{\pi_1}[\text{GeV}]$	1.6	1.8	2.0
$g_\rho[\text{GeV}^{-1}]$	$2.5 \sim 3.2$	$2.6 \sim 3.3$	$2.6 \sim 3.4$

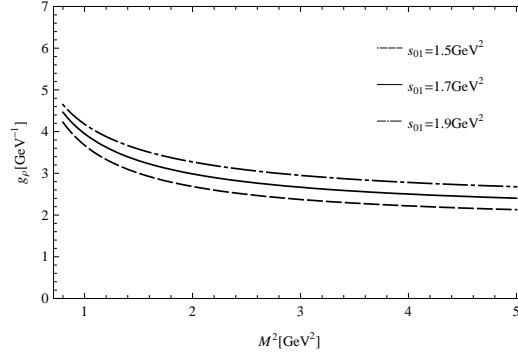


FIG. 2: The sum rule for g_ρ with $m_{\pi_1} = 1.6$ GeV, $2.3 < M^2 < 2.7$ GeV², and $s_{01} = 1.5, 1.7, 1.9$ GeV².

III. SUM RULES FOR THE $\pi_1 \rightarrow f_1 \pi, b_1 \pi$ COUPLING CONSTANTS

Using the method outlined above we can further derive the sum rules for the $\pi_1 \rightarrow f_1 \pi, b_1 \pi$ coupling constants. The effective Lagrangians of the process $\pi_1 \rightarrow f_1 \pi$ can be written as

$$\begin{aligned} \mathcal{L}_{f_1} = & g_{f_1}^1 \vec{\pi}_{1\alpha} \times \vec{f}_{1\beta} \cdot \vec{\pi} g^{\alpha\beta} \\ & + g_{f_1}^{21} \partial^\beta \vec{\pi}_{1\alpha} \times \partial^\alpha \vec{f}_{1\beta} \cdot \vec{\pi} + g_{f_1}^{22} \partial^\beta \vec{\pi}_{1\alpha} \times \vec{f}_{1\beta} \cdot \partial^\alpha \vec{\pi} \\ & + g_{f_1}^{23} \vec{\pi}_{1\alpha} \times \partial^\alpha \vec{f}_{1\beta} \cdot \partial^\beta \vec{\pi} + g_{f_1}^{24} \vec{\pi}_{1\alpha} \times \vec{f}_{1\beta} \cdot \partial^\alpha \partial^\beta \vec{\pi}. \end{aligned} \quad (15)$$

Then the decay amplitude for this process is

$$\mathcal{M}(\pi_1 \rightarrow f_1 + \pi) = i g_{f_1}^1 (\eta \cdot \epsilon^*) + i g_{f_1}^2 (\eta \cdot k)(\epsilon^* \cdot p), \quad (16)$$

where $g_{f_1}^2 = g_{f_1}^{21} - g_{f_1}^{22} + g_{f_1}^{23} - g_{f_1}^{24}$, ϵ^* is the polarization vector of the final f_1 meson, k and p are the momentum of the f_1 and the π_1 meson, respectively. The interpolating current for the f_1 meson is the axial-vector current

$$J_\mu^{f_1}(x) = \frac{1}{\sqrt{2}} [\bar{u}(x) \gamma_\mu \gamma_5 u(x) + \bar{d}(x) \gamma_\mu \gamma_5 d(x)]. \quad (17)$$

The f_1 meson couples to the above current through $\langle 0 | J_\mu^{f_1}(0) | f_1(k, \lambda) \rangle = f_{f_1} m_{f_1} \epsilon_\mu^\lambda$. We consider the following correlation functions:

$$\begin{aligned} \Pi_{f_1}(k^2, p^2) &= i \int d^4 x e^{ik \cdot x} \langle \pi(q) | T \{ J_\alpha^{f_1}(x) J_\beta^{\pi_1 \dagger}(0) \} | 0 \rangle \\ &= - \left(g_{\alpha\nu} - \frac{p_\alpha p_\nu}{m_{\pi_1}^2} \right) \left(g_{\beta\mu} - \frac{k_\beta k_\mu}{m_{f_1}^2} \right) \frac{\tilde{f}_{\pi_1} f_{f_1} m_{f_1}}{(m_{\pi_1}^2 - p^2)(m_{f_1}^2 - k^2)} \\ &\quad \times [g_{f_1}^1 g_{\mu\nu} + g_{f_1}^{21} k_\mu p_\nu + g_{f_1}^{22} q_\mu p_\nu + g_{f_1}^{23} k_\mu q_\nu + g_{f_1}^{24} q_\mu q_\nu] + \dots \\ &= - \left(g_{\alpha\nu} - \frac{p_\alpha p_\nu}{m_{\pi_1}^2} \right) \left(g_{\beta\mu} - \frac{k_\beta k_\mu}{m_{f_1}^2} \right) \frac{\tilde{f}_{\pi_1} f_{f_1} m_{f_1}}{(m_{\pi_1}^2 - p^2)(m_{f_1}^2 - k^2)} \\ &\quad \times [g_{f_1}^1 g_{\mu\nu} + g_{f_1}^{21} k_\mu p_\nu + g_{f_1}^{22} (p_\mu - k_\mu) p_\nu + g_{f_1}^{23} k_\mu (p_\nu - k_\nu) + g_{f_1}^{24} (p_\mu - k_\mu) (p_\nu - k_\nu)] + \dots \\ &= - \frac{\tilde{f}_{\pi_1} f_{f_1} m_{f_1}}{(m_{\pi_1}^2 - p^2)(m_{f_1}^2 - k^2)} [g_{f_1}^1 g_{\alpha\beta} + (g_{f_1}^{21} - g_{f_1}^{22} + g_{f_1}^{23} - g_{f_1}^{24}) p_\alpha k_\beta] + \dots \\ &= - \frac{\tilde{f}_{\pi_1} f_{f_1} m_{f_1}}{(m_{\pi_1}^2 - p^2)(m_{f_1}^2 - k^2)} [g_{f_1}^1 g_{\alpha\beta} + g_{f_1}^2 p_\alpha k_\beta] + \dots \end{aligned} \quad (18)$$

The axial-vector current (17) also couples to $I = 0$ pseudoscalar meson η/η' . However we can differentiate the contribution of the P -wave channel $\pi_1 \rightarrow \eta/\eta' \pi$ to the correlation function from that of $\pi_1 \rightarrow f_1 \pi$ due to their different Lorentz structures. In a similar way to Section. II, we obtain the following sum rules for $g_{f_1}^1$ and $g_{f_1}^2$ before the continuum subtraction:

$$f_{f_1} \tilde{f}_{\pi_1} m_{f_1} g_{f_1}^1 e^{\bar{u}_0 m_{f_1}^2 / M^2 + u_0 m_{\pi_1}^2 / M^2} + \dots$$

$$\begin{aligned}
&= \frac{f_\pi m_\pi^2}{2\sqrt{2}} \left\{ \left[\left(\frac{\partial \mathcal{A}_\perp}{\partial \alpha_3} - \frac{\partial \mathcal{A}_\perp}{\partial \alpha_2} \right)^{[\alpha_1]} + \left(\frac{\partial \mathcal{V}_\parallel}{\partial \alpha_3} - \frac{\partial \mathcal{V}_\parallel}{\partial \alpha_2} \right)^{[\alpha_1]} - \left(\frac{\partial \mathcal{V}_\perp}{\partial \alpha_3} - \frac{\partial \mathcal{V}_\perp}{\partial \alpha_2} \right)^{[\alpha_1]} - \left(\frac{\partial \mathcal{A}_\perp}{\partial \alpha_3} - \frac{\partial \mathcal{A}_\perp}{\partial \alpha_1} \right)^{[\alpha_2]} \right. \right. \\
&\quad + \left. \left(\frac{\partial \mathcal{V}_\parallel}{\partial \alpha_3} - \frac{\partial \mathcal{V}_\parallel}{\partial \alpha_1} \right)^{[\alpha_2]} - \left(\frac{\partial \mathcal{V}_\perp}{\partial \alpha_3} - \frac{\partial \mathcal{V}_\perp}{\partial \alpha_1} \right)^{[\alpha_2]} + \mathcal{A}_\perp(\bar{u}_0, u_0, 0) - \mathcal{A}_\perp(u_0, \bar{u}_0, 0) + \mathcal{V}_\parallel(\bar{u}_0, u_0, 0) + \mathcal{V}_\parallel(u_0, \bar{u}_0, 0) \right. \\
&\quad \left. \left. - \mathcal{V}_\perp(\bar{u}_0, u_0, 0) - \mathcal{V}_\perp(u_0, \bar{u}_0, 0) \right] M^4 + [\phi'_\pi(\bar{u}_0) - \phi'_\pi(u_0)] \frac{\langle g_s^2 G^2 \rangle}{m_\pi^2} M^2 + \int_{\bar{u}_0}^{u_0} \mathbb{B}(u) du \langle g_s^2 G^2 \rangle \right\}, \quad (19)
\end{aligned}$$

$$\begin{aligned}
&f_{f_1} \tilde{f}_{\pi_1} m_{f_1} g_{f_1}^2 e^{\bar{u}_0 m_{f_1}^2 / M^2 + u_0 m_{\pi_1}^2 / M^2} + \dots \\
&= \sqrt{2} f_\pi m_\pi^2 \left\{ \left[\mathcal{A}_\parallel^{[\alpha_2, \bar{\alpha}_2]} - \mathcal{A}_\parallel^{[\alpha_1, \bar{\alpha}_1]} + \mathcal{A}_\perp^{[\alpha_2, \bar{\alpha}_2]} - \mathcal{A}_\perp^{[\alpha_1, \bar{\alpha}_1]} - \mathcal{V}_\parallel^{[\alpha_2, \bar{\alpha}_2]} - \mathcal{V}_\parallel^{[\alpha_1, \bar{\alpha}_1]} - \mathcal{V}_\perp^{[\alpha_2, \bar{\alpha}_2]} - \mathcal{V}_\perp^{[\alpha_1, \bar{\alpha}_1]} \right. \right. \\
&\quad + \mathcal{A}_\parallel^{[\alpha_1, u_0]} - \mathcal{A}_\parallel^{[\alpha_2, u_0]} + \mathcal{A}_\perp^{[\alpha_1, u_0]} - \mathcal{A}_\perp^{[\alpha_2, u_0]} + \mathcal{V}_\parallel^{[\alpha_1, u_0]} + \mathcal{V}_\parallel^{[\alpha_2, u_0]} + \mathcal{V}_\perp^{[\alpha_1, u_0]} + \mathcal{V}_\perp^{[\alpha_2, u_0]} + \frac{1}{2}(\mathcal{V}_\perp^{[\alpha_1]} + \mathcal{V}_\perp^{[\alpha_2]}) \\
&\quad + \frac{1-4u_0}{2}(\mathcal{A}_\perp^{[\alpha_2]} - \mathcal{A}_\perp^{[\alpha_1]}) - \frac{1-2u_0}{2}(\mathcal{V}_\parallel^{[\alpha_1]} + \mathcal{V}_\parallel^{[\alpha_2]}) \left. \right] M^2 + \frac{\sqrt{2}}{2} u_0 [\phi_\pi(u_0) + \phi_\pi(\bar{u}_0)] \frac{\langle g_s^2 G^2 \rangle}{m_\pi^2} \\
&\quad + \frac{u_0 \bar{u}_0}{72} \int_{\bar{u}_0}^{u_0} \mathbb{B}(u) du \frac{\langle g_s^2 G^2 \rangle}{M^2} \left. \right\}. \quad (20)
\end{aligned}$$

Here we omitted the terms $\sim \mathcal{O}(m_\pi^4)$. After subtracting the continuum contribution, we get the sum rules for $g_{f_1}^1$ and $g_{f_1}^2$, which are plotted with $s_{01} = 2.0, 2.2, 2.4 \text{ GeV}^2$ in Figs. 3-4.

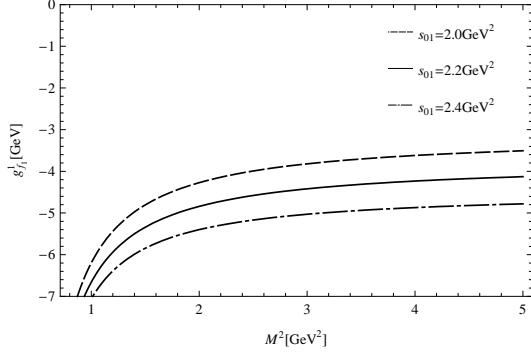


FIG. 3: The sum rule for $g_{f_1}^1$ with $m_{\pi_1} = 1.6 \text{ GeV}$, $1.7 < M^2 < 2.1 \text{ GeV}^2$, and $s_{01} = 2.0, 2.2, 2.4 \text{ GeV}^2$.

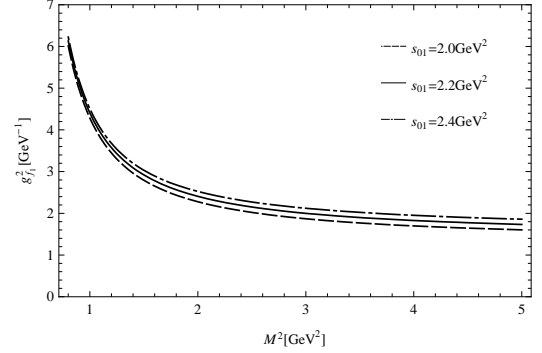


FIG. 4: The sum rule for $g_{f_1}^2$ with $m_{\pi_1} = 1.6 \text{ GeV}$, $2.3 < M^2 < 2.7 \text{ GeV}^2$, and $s_{01} = 2.0, 2.2, 2.4 \text{ GeV}^2$.

The derivation of the sum rules for $g_{b_1}^1$ and $g_{b_1}^2$ is almost the same as mentioned above. The interpolating current for the b_1 meson reads

$$J_\mu^{b_1}(x) = \bar{d}(x) \overleftrightarrow{\partial}_\mu \gamma_5 u(x), \quad (21)$$

where $\overleftrightarrow{\partial}_\mu \equiv \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu$, and we define $\langle 0 | J_\mu^{b_1}(0) | b_1(k, \lambda) \rangle = f_{b_1} \epsilon_\mu^\lambda$. There is another possible interpolating current for b_1 : $J'_\mu(x) = \bar{d}(x) \sigma_{\mu\nu} u(x)$, which couples to b_1 through $\langle 0 | J'_{\mu\nu}(0) | b_1(k, \lambda) \rangle = i f_{b_1}^T \varepsilon_{\mu\nu\rho\sigma} \epsilon_\lambda^\rho k^\sigma$. The same current couples to the ρ meson through $\langle 0 | J'_{\mu\nu}(0) | \rho(k, \lambda) \rangle = i f_\rho^T (\epsilon_\mu^\lambda k_\nu - \epsilon_\nu^\lambda k_\mu)$. Unfortunately, the Lorentz structure of these two coupling will mix with each other in the correlation function

$$\Pi'_{\mu\nu\beta}(k^2, p^2) = i \int d^4x e^{ik \cdot x} \langle \pi(q) | T \{ J'_{\mu\nu}(x) J_\beta^{\pi_1^\dagger}(0) \} | 0 \rangle, \quad (22)$$

so that we are unable to separate the contribution of the b_1 part from that of the ρ part. This is the consideration behind our choice of the interpolating current $J_\mu^{b_1}$ for the b_1 meson instead of the tensor one.

The sum rules for $g_{b_1}^1$ and $g_{b_1}^2$ read as

$$f_{b_1} \tilde{f}_{\pi_1} g_{b_1}^1 e^{\bar{u}_0 m_{b_1}^2 / M^2 + u_0 m_{\pi_1}^2 / M^2} + \dots = \frac{f_\pi m_\pi^2}{216\sqrt{2}(m_u + m_d)} [\phi'_\sigma(u_0) - \phi'_\sigma(\bar{u}_0)] M^2 \langle g_s^2 G^2 \rangle, \quad (23)$$

$$\begin{aligned}
& f_{b_1} \tilde{f}_{\pi_1} g_{b_1}^2 e^{\bar{u}_0 m_{b_1}^2 / M^2 + u_0 m_{\pi_1}^2 / M^2} + \dots \\
& = \frac{f_{\pi} m_{\pi}^2}{108\sqrt{2}(m_u + m_d)} \left\{ 216u_0 \left[\mathcal{T}(u_0, \bar{u}_0, 0) + \mathcal{T}(\bar{u}_0, u_0, 0) + \left(\frac{\partial \mathcal{T}}{\partial \alpha_3} - \frac{\partial \mathcal{T}}{\partial \alpha_2} \right)^{[\alpha_1]} + \left(\frac{\partial \mathcal{T}}{\partial \alpha_3} - \frac{\partial \mathcal{T}}{\partial \alpha_1} \right)^{[\alpha_2]} \right. \right. \\
& \quad \left. \left. - \mathcal{T}^{[\alpha_1]} - \mathcal{T}^{[\alpha_2]} \right] M^4 + \left[(1 - 2u_0)[\phi_{\sigma}(u_0) + \phi_{\sigma}(\bar{u}_0)] + u_0 \bar{u}_0 [\phi'_{\sigma}(u_0) - \phi'_{\sigma}(\bar{u}_0)] \right] \langle g_s^2 G^2 \rangle \right\}. \tag{24}
\end{aligned}$$

They are plotted in Fig. 5-6 after the continuum subtraction.

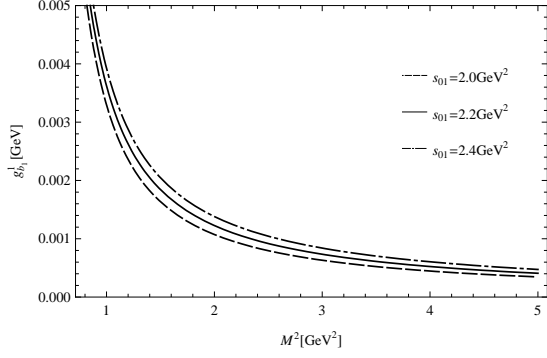


FIG. 5: The sum rule for $g_{b_1}^1$ with $m_{\pi_1} = 1.6$ GeV and $s_{01} = 2.0, 2.2, 2.4$ GeV². There is no stable working interval of M^2 for the sum rule.

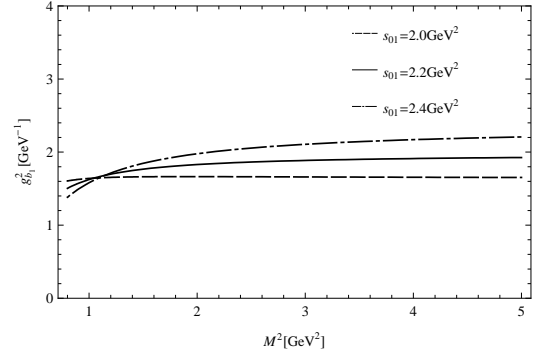


FIG. 6: The sum rule for $g_{b_1}^2$ with $m_{\pi_1} = 1.6$ GeV and $s_{01} = 2.0, 2.2, 2.4$ GeV². There is no stable working interval of M^2 for the sum rule.

The adopted values of the parameters $f_{f_1} = 0.17$ GeV and $f_{b_1} = 0.18$ GeV³ are obtained from Eq. (4.52) and Eq. (A.20) in Ref. [32], respectively. The extracted values of $g_{f_1}^1$, $g_{f_1}^2$, $g_{b_1}^1$, and $g_{b_1}^2$ are collected in Table I.

m_{π_1} [GeV]	1.6	1.8	2.0	M^2 [GeV ²]
$g_{f_1}^1$ [GeV]	-4.2 ~ -5.6	-4.4 ~ -5.9	-4.6 ~ -6.1	1.7 ~ 2.1
$g_{f_1}^2$ [GeV ⁻¹]	2.0 ~ 2.4	2.1 ~ 2.5	2.3 ~ 2.7	2.3 ~ 2.7
$g_{b_1}^1$ [GeV]	$\ll 0.1$	$\ll 0.1$	$\ll 0.1$	-
$g_{b_1}^2$ [GeV ⁻¹]	1.9	1.8	1.6	-

TABLE I: The numerical values of $g_{f_1}^1$, $g_{f_1}^2$, $g_{b_1}^1$, and $g_{b_1}^2$. The working intervals of the Borel parameter M^2 are listed in the right column. Here “-” indicates the nonexistence of a stable working interval for the corresponding sum rule.

The values presented in these cases are determined by $M^2 = 2.0$ GeV². The range of the threshold is $2.0 < s_{01} < 2.4$ GeV².

IV. SUM RULES FOR THE $\pi_1 \rightarrow \eta\pi, \eta'\pi$ COUPLING CONSTANTS

The interpolating currents for η and η' read as

$$\begin{aligned}
J^{\eta} &= J^{\eta s} \cos \theta - J^{\eta_1} \sin \theta, \\
J^{\eta'} &= J^{\eta s} \sin \theta + J^{\eta_1} \cos \theta,
\end{aligned} \tag{25}$$

where

$$\begin{aligned}
J^{\eta s}(x) &= \frac{1}{\sqrt{6}} [\bar{u}(x) i\gamma_5 u(x) + \bar{d}(x) i\gamma_5 d(x) - 2\bar{s}(x) i\gamma_5 s(x)], \\
J^{\eta_1}(x) &= \frac{1}{\sqrt{3}} [\bar{u}(x) i\gamma_5 u(x) + \bar{d}(x) i\gamma_5 d(x) + \bar{s}(x) i\gamma_5 s(x)],
\end{aligned} \tag{26}$$

and $\theta = -19^\circ$ is the mixing angle between η_8 and the SU(3) singlet η_1 . Their coupling to η and η' are $\langle 0|J^\eta(0)|\eta(k)\rangle = \lambda_\eta$ and $\langle 0|J^{\eta'}(0)|\eta'(k)\rangle = \lambda_{\eta'}$, respectively. The correlation function involved in our calculation is

$$\Pi_{\eta/\eta'}(k^2, p^2) = i \int d^4x e^{ik \cdot x} \langle \pi(q) | T \{ J^{\eta/\eta'}(x) J_{\beta}^{\pi_1 \dagger}(0) \} | 0 \rangle = q^\beta G_{\eta/\eta'}(k^2, p^2) + \dots, \quad (27)$$

The sum rules derived for g_η and $g_{\eta'}$ are

$$\begin{aligned} & \lambda_\eta \tilde{f}_{\pi_1} g_\eta e^{\bar{u}_0 m_\eta^2/M^2 + u_0 m_{\pi_1}^2/M^2} + \dots \\ &= \frac{\cos \theta f_\pi m_\pi^2}{216\sqrt{2}(m_u + m_d)} \left\{ 216 \left[\left(\frac{\partial \mathcal{T}}{\partial \alpha_3} - \frac{\partial \mathcal{T}}{\partial \alpha_2} \right)^{[\alpha_1]} + \left(\frac{\partial \mathcal{T}}{\partial \alpha_3} - \frac{\partial \mathcal{T}}{\partial \alpha_1} \right)^{[\alpha_2]} + \mathcal{T}(u_0, \bar{u}_0, 0) + \mathcal{T}(\bar{u}_0, u_0, 0) \right] M^4 \right. \\ & \quad \left. + \bar{u}_0 [\phi'_\sigma(u_0) - \phi'_\sigma(\bar{u}_0)] \langle g_s^2 G^2 \rangle \right\}, \end{aligned} \quad (28)$$

$$\begin{aligned} & \lambda_{\eta'} \tilde{f}_{\pi_1} g_{\eta'} e^{\bar{u}_0 m_{\eta'}^2/M^2 + u_0 m_{\pi_1}^2/M^2} + \dots \\ &= \frac{\sin \theta f_\pi m_\pi^2}{216\sqrt{2}(m_u + m_d)} \left\{ 216 \left[\left(\frac{\partial \mathcal{T}}{\partial \alpha_3} - \frac{\partial \mathcal{T}}{\partial \alpha_2} \right)^{[\alpha_1]} + \left(\frac{\partial \mathcal{T}}{\partial \alpha_3} - \frac{\partial \mathcal{T}}{\partial \alpha_1} \right)^{[\alpha_2]} + \mathcal{T}(u_0, \bar{u}_0, 0) + \mathcal{T}(\bar{u}_0, u_0, 0) \right] M^4 \right. \\ & \quad \left. + \bar{u}_0 [\phi'_\sigma(u_0) - \phi'_\sigma(\bar{u}_0)] \langle g_s^2 G^2 \rangle \right\}. \end{aligned} \quad (29)$$

We take $m_\eta = 0.547$ GeV, $\lambda_\eta = 0.23$ GeV², $m_{\eta'} = 0.958$ GeV, and $\lambda_{\eta'} = 0.33$ GeV² in our numerical analysis [33]. The extracted values of g_η and $g_{\eta'}$ are presented here:

m_{π_1} [GeV]	1.6	1.8	2.0	M^2 [GeV ²]
g_η	0.45	0.45	0.42	-
$g_{\eta'}$	-0.16	-0.15	-0.15	-

V. PARTIAL DECAY WIDTHS

It is straightforward to calculate the partial widths of π_1 with the extracted coupling constants. The formulae for these partial widths are given in Eq. (30).

$$\begin{aligned} \Gamma_{\pi_1^0 \rightarrow \rho^+ \pi^-} &= \frac{g_\rho^2}{96\pi m_{\pi_1}^3} |\vec{q}_\pi|^3, \\ \Gamma_{\pi_1^0 \rightarrow f_1 \pi^0} &= \frac{1}{24\pi m_{\pi_1}^2} \left[(g_{f_1}^1)^2 (3 + \frac{|\vec{q}_\pi|^2}{m_{f_1}^2}) |\vec{q}_\pi| + (g_{f_1}^2)^2 \frac{m_{\pi_1}^2}{m_{f_1}^2} |\vec{q}_\pi|^5 + 2g_{f_1}^1 g_{f_1}^2 \frac{m_{\pi_1}}{m_{f_1}^2} \sqrt{m_{f_1}^2 + |\vec{q}_\pi|^2} |\vec{q}_\pi|^3 \right], \\ \Gamma_{\pi_1^0 \rightarrow b_1^+ \pi^-} &= \Gamma_{\pi_1^0 \rightarrow f_1 \pi^0} (g_{f_1}^1 \rightarrow g_{b_1}^1, g_{f_1}^2 \rightarrow g_{b_1}^2, m_{f_1} \rightarrow m_{b_1}), \\ \Gamma_{\pi_1^0 \rightarrow \eta \pi^0} &= \frac{g_\eta^2}{192\pi m_{\pi_1}^3} |\vec{q}_\pi|^3, \\ \Gamma_{\pi_1^0 \rightarrow \eta' \pi^0} &= \Gamma_{\pi_1^0 \rightarrow \eta \pi^0} (g_\eta \rightarrow g_{\eta'}, m_\eta \rightarrow m_{\eta'}). \end{aligned} \quad (30)$$

For completeness, we make rough estimates of the decay widths of some decay modes with the given coupling constants, although the corresponding coupling constants are extracted from sum rules without a stable working interval. We collected our results in Table II, together with the results obtained using other phenomenological models, e.g. Ref. [19] (PSS) and Ref. [18] (IKP). Here we also reproduce Table XIII of Ref. [35] to present the existing experimental results on the total decay width of π_1 in Table III in order to compare with our predictions.

The numerical values of the coupling constants of the modes $\pi_1 \rightarrow \rho\pi, f_1\pi$ are stable with the variation of m_{π_1} . As a consequence, the partial widths of these two modes increase rapidly with m_{π_1} due to their enlarged two-body phase spaces.

Apparently, our results on the partial width of π_1 are quite different from those obtained using Lattice QCD [40] and other phenomenological approaches such as the IKP and PSS flux tube model. In the flux tube model, the π_1

$m_{\pi_1} [\text{GeV}]$	1.6			1.8			2.0			
	IKP [18]	PSS [19]	this work	IKP [18]	PSS [19]	this work	IKP [18]	PSS [19]	Lattice [40]	this work
$\rho\pi$	8	9	$73 \sim 120$	12	13	$138 \sim 222$		16		$216 \sim 370$
$f_1\pi$	14	5	$69 \sim 122$	21	9	$96 \sim 175$		10.2	90 ± 60	$109 \sim 195$
$b_1\pi$	59	24	0.14	62	38	1.2		43	400 ± 120	3.7
$\eta\pi$	0	0	0.36	.02	.02	0.44		.02		0.45
$\eta'\pi$	0	0	0.02	0	.01	0.02		.01		0.03

TABLE II: The partial widths of the single-pion channels of π_1 in units of MeV, where IKP and PSS refer to the methods used in Ref. [18] and Ref. [19], respectively. The partial widths for the channels $b_1\pi$ and $f_1\pi$ cited from IKP and PSS are simply the sum of their S -wave and D -wave widths.

Mode	Mass (GeV)	Width (GeV)	Experiment	Reference
$\rho\pi$	1.593 ± 0.08	0.168 ± 0.020	E852	[5]
$\eta'\pi$	1.597 ± 0.010	0.340 ± 0.040	E852	[6]
$f_1\pi$	1.709 ± 0.024	0.403 ± 0.080	E852	[7]
$b_1\pi$	1.664 ± 0.008	0.185 ± 0.025	E852	[8]
$b_1\pi$	1.58 ± 0.03	0.30 ± 0.03	VES	[36]
$b_1\pi$	1.61 ± 0.02	0.290 ± 0.03	VES	[4]
$b_1\pi$	~ 1.6	~ 0.33	VES	[37]
$b_1\pi$	1.56 ± 0.06	0.34 ± 0.06	VES	[38]
$f_1\pi$	1.64 ± 0.03	0.24 ± 0.06	VES	[38]
$\eta'\pi$	1.58 ± 0.03	0.30 ± 0.03	VES	[36]
$\eta'\pi$	1.61 ± 0.02	0.290 ± 0.03	VES	[37]
$\eta'\pi$	1.56 ± 0.06	0.34 ± 0.06	VES	[38]
$b_1\pi$	~ 1.6	~ 0.23	CBAR	[9]
$\rho\pi$	1.660 ± 0.010	0.269 ± 0.021	COMPASS	[1]
all	$1.662^{+0.015}_{-0.011}$	0.234 ± 0.050	PDG	[39]

TABLE III: Reported masses and widths of the $\pi_1(1600)$ from the E852 experiment, the VES experiment and the COMPASS experiment. The PDG average from 2008 is also reported. (Table reproduced from Ref. [35].)

coupling to the two S -wave mesons is suppressed, leading to a small branch ratio of the mode $\pi_1 \rightarrow \rho\pi$. One flux tube model prediction [41] for widths for π_1 with $m_{\pi_1} = 2.0$ GeV is (in MeV)

$$\pi f_1 : \pi b_1 : \pi \rho : \eta\pi : \eta'\pi = 60 : 170 : 5 \sim 20 : 0 \sim 10 : 0 \sim 10. \quad (31)$$

The quenched lattice QCD simulation also predicted a large width of the channel $b_1\pi$ [40], although the linear extrapolation approximation adopted there may lead to an overestimated width for this channel, as pointed out in Ref. [20]. However, the $b_1\pi$ channel is severely suppressed in our calculation since the numerical value of $g_{b_1}^1$ is found to be extremely small. As far as the channels $\eta\pi$ and $\eta'\pi$ are concerned, Chung, Klempt, and Korner argued that the channel $\pi_1 \rightarrow \eta\pi$ is forbidden due to the requirement of Bose symmetry and J^{PC} conservation in the limit that the η is a pure SU(3) octet [42]. The tiny mixing between η_8 and η_1 should not reverse the widths of these two channels. In contrast, the width of $\eta\pi$ is at least one order of magnitude larger than that of the channel $\eta'\pi$ in our calculation. Experimentally, the relative branching ratios for $\pi_1(1600)$'s three channels $b_1\pi$, $\eta'\pi$, and $\rho\pi$ are [4]

$$b_1\pi : \eta'\pi : \rho\pi = 1 : 1 \pm 0.3 : 1.5 \pm 0.5. \quad (32)$$

In a summary on the VES results, Amelin [38] obtained the relative branch ratios for the $\pi_1(1600)$ as follows:

$$b_1\pi : f_1\pi : \rho\pi : \eta'\pi = 1.0 \pm .3 : 1.1 \pm .3 : < .3 : 1. \quad (33)$$

VI. STRONG DECAYS OF THE ISOSCALAR 1^{-+} HYBRID STATE

Now we consider the strong decays of $\tilde{\pi}_1$, the isoscalar partner of π_1 . We notice that $I^G J^{PC}$ conservation restricts the possible decay channels to the S -wave $\tilde{\pi}_1 \rightarrow a_1(1260)\pi$, $f_1(1285)\eta$, and the P -wave $\tilde{\pi}_1 \rightarrow \eta\eta'$, $\pi(1300)\pi$, $\eta(1295)\eta$. The partial widths of the three P -wave channels are supposed to be relatively small due to their small phase spaces. In addition, the channel $\tilde{\pi}_1 \rightarrow f_1\eta$ is kinematically forbidden if the mass of $\tilde{\pi}_1$ is smaller than 1.83 GeV. Hence the dominant decay mode of $\tilde{\pi}_1$ is $\tilde{\pi}_1 \rightarrow a_1\pi$.

The interpolating currents for the $\tilde{\pi}_1$ meson and the a_1 meson are

$$\begin{aligned} J_{\mu}^{\tilde{\pi}_1}(x) &= \frac{1}{\sqrt{2}} \left[\bar{u}(x) \frac{\lambda^a}{2} g_s G_{\mu\nu}^a(x) \gamma^{\nu} u(x) + \bar{d}(x) \frac{\lambda^a}{2} g_s G_{\mu\nu}^a(x) \gamma^{\nu} d(x) \right], \\ J_{\mu}^{a_1}(x) &= \frac{1}{\sqrt{2}} [\bar{u}(x) \gamma_{\mu} \gamma_5 u(x) - \bar{d}(x) \gamma_{\mu} \gamma_5 d(x)]. \end{aligned} \quad (34)$$

We define the coupling constants $g_{a_1}^1$ and $g_{a_1}^2$, similar to the coupling constants $g_{f_1}^1$ and $g_{f_1}^2$ of the channel $\pi_1 \rightarrow f_1\pi$. Apparently, the sum rules for $g_{a_1}^1$ and $g_{a_1}^2$ before the continuum subtraction are similar to Eq. (19) and (20), respectively. The only difference between them lies in the hadron-level parameters, namely the masses and the overlapping amplitudes of the mesons involved in these two decay modes. Hence it is plausible to write down $g_{a_1}^1 \approx g_{f_1}^1$ and $g_{a_1}^2 \approx g_{f_1}^2$ if we simply adopt $\tilde{f}_{\tilde{\pi}_1} = \tilde{f}_{\pi_1} = 0.15 \text{ GeV}^4$ and $f_{a_1} = f_{f_1} = 0.17 \text{ GeV}$ in our numerical analysis. This leads to the estimate of the partial width of the mode $\tilde{\pi}_1 \rightarrow a_1\pi$: $\Gamma_{\tilde{\pi}_1 \rightarrow a_1\pi} \approx 3\Gamma_{\pi_1 \rightarrow f_1\pi}$. Here the factor 3 comes from the difference between the two channels' final states, namely $\tilde{\pi}_1 \rightarrow a_1^+\pi^-, a_1^-\pi^+, a_1^0\pi^0$ versus $\pi_1^0 \rightarrow f_1\pi^0$.

If we denote the right side of the sum rules for $g_{f_1}^1$ in Eq. (19) and $g_{f_1}^2$ in Eq. (20) as $R_{f_1}^1$ and $R_{f_1}^2$, respectively, then we have the following sum rules for $g_{f_1\eta}^1$ and $g_{f_1\eta}^2$:

$$\begin{aligned} f_{f_1} \tilde{f}_{\tilde{\pi}_1} m_{f_1} g_{f_1\eta}^1 e^{\bar{u}_0 m_{f_1}^2/M^2 + u_0 m_{\tilde{\pi}_1}^2/M^2} + \dots &= \frac{1}{\sqrt{3}} R_{f_1}^1(\pi \rightarrow \sigma, f_{\pi} \rightarrow f_{\sigma}, m_{\pi} \rightarrow m_{\sigma}, \dots), \\ f_{f_1} \tilde{f}_{\tilde{\pi}_1} m_{f_1} g_{f_1\eta}^2 e^{\bar{u}_0 m_{f_1}^2/M^2 + u_0 m_{\tilde{\pi}_1}^2/M^2} + \dots &= \frac{1}{\sqrt{3}} R_{f_1}^2(\pi \rightarrow \sigma, f_{\pi} \rightarrow f_{\sigma}, m_{\pi} \rightarrow m_{\sigma}, \dots), \end{aligned} \quad (35)$$

where we have ignored the SU(3) singlet octet mixing in the $\eta\eta'$ system and the factor $e^{u_0 \bar{u}_0 m_{\eta}^2/M^2}$ because $m_{\eta}^2/M^2 < 0.1$ in our calculations.

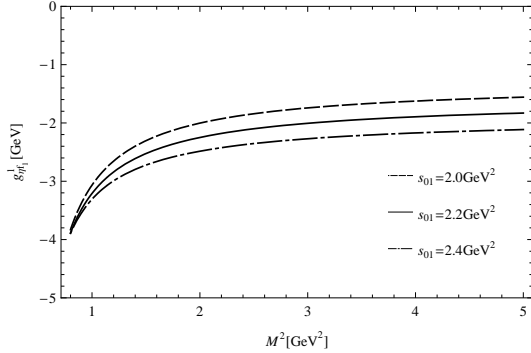


FIG. 7: The sum rule for $g_{f_1\eta}^1$ with $m_{\tilde{\pi}_1} = 2.0 \text{ GeV}$, $2.6 < M^2 < 3.0 \text{ GeV}^2$, and $s_{01} = 2.0, 2.2, 2.4 \text{ GeV}^2$.

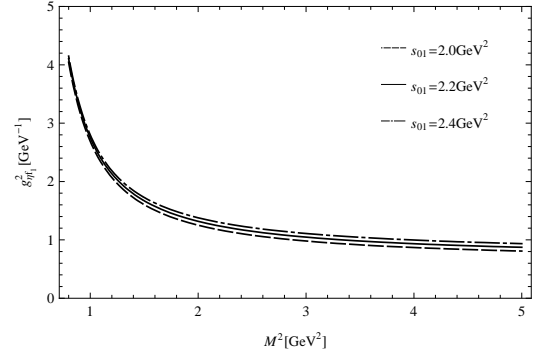


FIG. 8: The sum rule for $g_{f_1\eta}^2$ with $m_{\tilde{\pi}_1} = 2.0 \text{ GeV}$, $2.6 < M^2 < 3.0 \text{ GeV}^2$, and $s_{01} = 2.0, 2.2, 2.4 \text{ GeV}^2$.

We adopt $f_{\eta} = 130 \text{ MeV}$ and $\mu_{\eta} = 1.47 \text{ GeV}$ [31]. The mass of strange quark is taken to be $m_s = 0.15 \text{ GeV}$. The input parameters for the η light-cone distribution amplitudes involved in our calculation are as follows ($\mu = 1 \text{ GeV}$) [31]:

a_2	η_3	ω_3	η_4	ω_4	h_{00}	v_{00}	a_{10}	v_{10}	h_{01}	h_{10}
0.2	0.013	-3	0.5	0.2	-0.17	-0.17	0.17	0.26	0.15	0.38

The sum rules after the continuum subtraction are plotted in Fig. 7-8. Their numerical values for $2.0 < s_{01} < 2.4 \text{ GeV}^2$ are given here:

$m_{\tilde{\pi}_1}[\text{GeV}]$	1.6	1.8	2.0	$M^2[\text{GeV}^2]$
$g_{f_1\eta}^1[\text{GeV}]$	$-1.6 \sim -2.2$	$-1.7 \sim -2.3$	$-1.7 \sim -2.3$	$2.6 \sim 3.0$
$g_{f_1\eta}^2[\text{GeV}^{-1}]$	$0.8 \sim 1.0$	$0.9 \sim 1.1$	$1.0 \sim 1.2$	$2.6 \sim 3.0$

Our calculation of $\tilde{\pi}_1$'s widths are straightforward if we take advantage of the width formulae for $\pi_1 \rightarrow f_1\pi$:

$$\begin{aligned}\Gamma_{\tilde{\pi}_1^0 \rightarrow a_1^+ \pi^-} &= \Gamma_{\pi_1^0 \rightarrow f_1 \pi^0} (g_{f_1}^1 \rightarrow g_{a_1}^1, g_{f_1}^2 \rightarrow g_{a_1}^2, m_{f_1} \rightarrow m_{a_1}, m_{\pi_1} \rightarrow m_{\tilde{\pi}_1}), \\ \Gamma_{\tilde{\pi}_1^0 \rightarrow f_1 \eta} &= \Gamma_{\pi_1^0 \rightarrow f_1 \pi^0} (g_{f_1}^1 \rightarrow g_{f_1 \eta}^1, g_{f_1}^2 \rightarrow g_{f_1 \eta}^2, m_{\pi_1} \rightarrow m_{\tilde{\pi}_1}, m_{\pi} \rightarrow m_{\eta}).\end{aligned}\quad (36)$$

The partial widths of $\tilde{\pi}_1$ are listed in Table IV, along with the results obtained using other approaches. As mentioned above, the only dominant channel for $\tilde{\pi}_1$ is $\tilde{\pi}_1 \rightarrow a_1\pi$ whose width varies from 200 MeV to 600 MeV, depending heavily on the mass of $\tilde{\pi}_1$ we adopt.

$m_{\tilde{\pi}_1}[\text{GeV}]$	1.6			1.8			2.0		
	IKP PSS this work			IKP PSS this work			IKP PSS this work		
	[18]	[19]		[18]	[19]		[18]	[19]	
$a_1\pi$			207 ~ 366	72	28.2	288 ~ 525	30.6	327 ~ 585	
$f_1\eta$	-	-	-	-	-	-	8	10 ~ 19	

TABLE IV: The partial widths of $\tilde{\pi}_1$ in units of MeV, where “-” indicates that the corresponding channel is kinematically forbidden.

VII. SEARCH FOR THE 1^{-+} STATE AT BESIII

Since the BESIII detector has an excellent photon resolution, it's very interesting to search for the 1^{-+} state in the J/ψ radiative process $J/\psi(\psi') \rightarrow h_{1,0} + \gamma$. The photon spectrum peaks around $E_\gamma = \frac{m_{J/\psi}^2 - m_h^2}{2m_{J/\psi}}$ with a width $\frac{m_h \Gamma_h}{m_{J/\psi}} \sim (100 - 200) \text{ MeV}$. Such a process may be described by the following effective Lagrangian

$$\mathcal{L} = c_0 F_{\mu\nu} \psi^\mu h_{1,0}^\nu + c'_0 F_{\mu\nu} \psi^{\nu\alpha} h_{1,0}^\mu \quad (37)$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor, ψ_μ and $h_{1,0}^\mu$ are the J/ψ and 1^{-+} field. Naively one expects the above branching ratio to be around $10^{-5} \sim 10^{-4}$.

The isovector 1^{-+} state π_1 can also be produced associated with other hadrons X at BESIII through the process $J/\psi(\psi') \rightarrow \pi_1 + X$. For the production of the neutral component of π_1 , the quantum numbers of X are $I^G = 1^+, C = -$. Moreover, $m_X \leq m_{J/\psi} - m_{\pi_1} \sim 1.5 \text{ GeV}$. From the above constraint, we get $X = \rho^0, b_1^0, \rho(1450)$ if it is a single resonance or $X = \pi^+\pi^-$ etc. Let's focus on the case $X = \rho$. Such a production may be described by the following effective Lagrangian

$$\mathcal{L} = c_1 \psi_{\mu\nu} \vec{h}_1^\mu \cdot \vec{\rho}^\nu + c_2 \psi_\mu \vec{h}_1^{\mu\nu} \cdot \vec{\rho}^\nu + c_3 \psi_\mu \vec{h}_1^\nu \cdot \vec{\rho}^{\mu\nu} + c_4 \psi_{\mu\nu} \vec{h}_1^{\nu\alpha} \cdot \vec{\rho}_\alpha^\mu \quad (38)$$

Naively one expects the above branching ratio to be around $10^{-4} \sim 10^{-3}$.

From Table II, the dominant decay modes of the isovector 1^{-+} meson are $\rho\pi, f_1\pi$. We urge our BESIII colleagues to search for π_1 through the decay chain: $J/\psi(\psi') \rightarrow \pi_1 + \gamma, \pi_1 \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$ or $\pi_1 \rightarrow f_1(1285)\pi^0$. $f_1(1285)$ is a narrow state with a width of 24.3 MeV. The $f_1\pi^0$ mode is also useful in the search of π_1 , although $f_1(1285)$ mainly decays into multiple particle final states $4\pi, \eta\pi\pi$. The other important decay chain is $J/\psi(\psi') \rightarrow \pi_1 + \rho \rightarrow \rho + \rho + \pi \rightarrow 2(\pi^+\pi^-\pi^0)$. Once enough data are accumulated, one may also try to look for π_1 in the $b_1\pi, \eta\pi, \eta'\pi$ modes.

The isoscalar 1^{-+} state $\tilde{\pi}_1$ can also be produced associated with other hadrons X' at BESIII through the process $J/\psi(\psi') \rightarrow \tilde{\pi}_1 + X'$. Now the quantum numbers of X' are $I^G = 0^-, C = -$. The possible candidates are $X' = \omega, \phi, h_1(1170), \omega(1470), \pi^+\pi^-\pi^0$ etc. The $\tilde{\pi}_1$ state mainly decays into $a_1\pi$. Search of $\tilde{\pi}_1$ through the hadronic decay chain $J/\psi(\psi') \rightarrow \tilde{\pi}_1 + \omega/\phi \rightarrow a_1 + \pi + \omega/\phi$ is challenging since it involves too many pions in the final states. BESIII collaboration may also search for $\tilde{\pi}_1$ through the radiative decay chain: $J/\psi(\psi') \rightarrow \tilde{\pi}_1 + \gamma \rightarrow a_1 + \pi + \gamma$.

VIII. CONCLUSION

We have studied the major strong decay modes of the $J^{PC} = 1^{-+}$ hybrid mesons, including the isovector and the isoscalar cases. The coupling constants for these modes are extracted with the Light-cone QCD sum rule approach. Most of the sum rules obtained are stable with the variations of the Borel parameter M^2 and the continuum threshold s_{01} . For the other sum rules, we can not find a stable working interval of M^2 .

Some possible sources of the errors in our calculation include the inherent inaccuracy of LCQSR: the omission of the higher twist terms in the OPE near the light-cone, the variation of the coupling constant with the continuum threshold s_{01} and the Borel parameter M^2 in the working interval, the omission of the higher conformal partial waves in the light-cone distribution amplitudes of the pion or the η , and the uncertainty in the parameters that appear in these light-cone distribution amplitudes. The uncertainty in the overlapping amplitudes between the interpolating currents and the corresponding final mesons is another source of errors. We merely give an estimated range for each involved coupling constant. The uncertainty of f_h , which was not taken into consideration in this work, may broaden these ranges significantly. It's also understood that the α_s correction may turn out to be quite large. We also omit the $\mathcal{O}(m_\pi^4)$ terms in the derivation of the sum rules for most of the coupling constants involved in our calculation.

The partial widths of the 1^{-+} hybrid calculated from the coupling constants extracted here are quite different from those obtained using other methods like the flux tube model and Lattice QCD etc. So far, the experimental data on the decay pattern of the 1^{-+} hybrid is still not so accurate. We suggest the possible search of the isovector and the isoscalar 1^{-+} hybrids in $J/\psi(\psi')$ decay processes at BESIII.

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Appendix A: The light-cone distribution amplitudes of the pion

The 2-particle distribution amplitudes of the π meson are defined as [31]

$$\begin{aligned}
 \langle 0 | \bar{u}(z) \gamma_\mu \gamma_5 d(-z) | \pi^-(P) \rangle &= i f_\pi p_\mu \int_0^1 du e^{i\xi pz} \phi_\pi(u) + \frac{i}{2} f_\pi m^2 \frac{1}{pz} z_\mu \int_0^1 du e^{i\xi pz} g_\pi(u), \\
 \langle 0 | \bar{u}(z) i \gamma_5 d(-z) | \pi(P) \rangle &= \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du e^{i\xi pz} \phi_p(u), \\
 \langle 0 | \bar{u}(z) \sigma_{\alpha\beta} \gamma_5 d(-z) | \pi(P) \rangle &= -\frac{i}{3} \frac{f_\pi m_\pi^2}{m_u + m_d} (p_\alpha z_\beta - p_\beta z_\alpha) \int_0^1 du e^{i\xi pz} \phi_\sigma(u),
 \end{aligned} \tag{A1}$$

where $\xi \equiv 2u - 1$, ϕ_π is the leading twist-2 distribution amplitude, $\phi_{(p,\sigma)}$ are of twist-3. All the above distribution amplitudes $\phi = \{\phi_\pi, \phi_p, \phi_\sigma, g_\pi\}$ are normalized to unity: $\int_0^1 du \phi(u) = 1$.

There is one 3-particle distribution amplitudes of twist-3, defined as [31]

$$\langle 0 | \bar{u}(z) \sigma_{\mu\nu} \gamma_5 g_s G_{\alpha\beta}(vz) d(-z) | \pi^-(P) \rangle = i \frac{f_\pi m_\pi^2}{m_u + m_d} (p_\alpha p_\mu g_{\nu\beta}^\perp - p_\alpha p_\nu g_{\mu\beta}^\perp - p_\beta p_\mu g_{\nu\alpha}^\perp + p_\beta p_\nu g_{\mu\alpha}^\perp) \mathcal{T}(v, pz), \tag{A2}$$

where we used the following notation for the integral defining the 3-particle distribution amplitude:

$$\mathcal{T}(v, pz) = \int \mathcal{D}\underline{\alpha} e^{-ipz(\alpha_u - \alpha_d + v\alpha_g)} \mathcal{T}(\alpha_d, \alpha_u, \alpha_g). \tag{A3}$$

Here $\underline{\alpha}$ is the set of three momentum fractions α_d , α_u , and α_g . The integration measure is

$$\int \mathcal{D}\underline{\alpha} = \int_0^1 d\alpha_d d\alpha_u d\alpha_g \delta(1 - \alpha_u - \alpha_d - \alpha_g). \tag{A4}$$

The 3-particle distribution amplitudes of twist-4 are

$$\begin{aligned}\langle 0|\bar{u}(z)\gamma_\mu\gamma_5g_sG_{\alpha\beta}(vz)d(-z)|\pi^-(P)\rangle &= p_\mu(p_\alpha z_\beta - p_\beta z_\alpha)\frac{1}{pz}f_\pi m_\pi^2\mathcal{A}_\parallel(v,pz) + (p_\beta g_{\alpha\mu}^\perp - p_\alpha g_{\beta\mu}^\perp)f_\pi m_\pi^2\mathcal{A}_\perp(v,pz), \\ \langle 0|\bar{u}(z)\gamma_\mu ig_s\tilde{G}_{\alpha\beta}(vz)d(-z)|\pi^-(P)\rangle &= p_\mu(p_\alpha z_\beta - p_\beta z_\alpha)\frac{1}{pz}f_\pi m_\pi^2\mathcal{V}_\parallel(v,pz) + (p_\beta g_{\alpha\mu}^\perp - p_\alpha g_{\beta\mu}^\perp)f_\pi m_\pi^2\mathcal{V}_\perp(v,pz),\end{aligned}\quad (\text{A5})$$

where $\tilde{G}_{\alpha\beta}$ is the dual field $\tilde{G}_{\alpha\beta} \equiv \frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}G^{\gamma\delta}$.

We also use the distribution amplitude given in Ref. [31]:

$$\phi_\pi(u) = 6u(1-u)\left(1 + a_2 C_2^{3/2}(\xi)\right), \quad (\text{A6})$$

$$g_\pi(u) = 1 + \left(1 + \frac{18}{7}a_2 + 60\eta_3 + \frac{20}{3}\eta_4\right)C_2^{1/2}(\xi) + \left(-\frac{9}{28}a_2 - 6\eta_3\omega_3\right)C_4^{1/2}(\xi), \quad (\text{A7})$$

$$\begin{aligned}\mathbb{A}(u) &= 6u\bar{u}\left\{\frac{16}{15} + \frac{24}{35}a_2 + 20\eta_3 + \frac{20}{9}\eta_4\right. \\ &\quad + \left(-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3\omega_3 - \frac{10}{27}\eta_4\right)C_2^{3/2}(\xi) + \left(-\frac{11}{210}a_2 - \frac{4}{135}\eta_3\omega_3\right)C_4^{3/2}(\xi)\Big\} \\ &\quad + \left(-\frac{18}{5}a_2 + 21\eta_4\omega_4\right)\{2u^3(10 - 15u + 6u^2)\ln u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln \bar{u} + u\bar{u}(2 + 13u\bar{u})\},\end{aligned}\quad (\text{A8})$$

$$\mathbb{B}(u) = g_\pi(u) - \phi_\pi(u), \quad (\text{A9})$$

$$\phi_p(u) = 1 + \left(30\eta_3 - \frac{5}{2}\rho_\pi^2\right)C_2^{1/2}(\xi) + \left(-3\eta_3\omega_3 - \frac{27}{20}\rho_\pi^2 - \frac{81}{10}\rho_\pi^2 a_2\right)C_4^{1/2}(\xi), \quad (\text{A10})$$

$$\phi_\sigma(u) = 6u(1-u)\left\{1 + \left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_\pi^2 - \frac{3}{5}\rho_\pi^2 a_2\right)C_2^{3/2}(\xi)\right\}, \quad (\text{A11})$$

$$\mathcal{T}(\underline{\alpha}) = 360\eta_3\alpha_u\alpha_d\alpha_g^2\left\{1 + \omega_3\frac{1}{2}(7\alpha_g - 3)\right\}, \quad (\text{A12})$$

$$\begin{aligned}\mathcal{V}_\parallel(\underline{\alpha}) &= 120\alpha_u\alpha_d\alpha_g(v_{00} + v_{10}(3\alpha_g - 1)), \\ \mathcal{A}_\parallel(\underline{\alpha}) &= 120\alpha_u\alpha_d\alpha_g a_{10}(\alpha_d - \alpha_u),\end{aligned}\quad (\text{A13})$$

$$\mathcal{V}_\perp(\underline{\alpha}) = -30\alpha_g^2\left[h_{00}(1 - \alpha_g) + h_{01}\left[\alpha_g(1 - \alpha_g) - 6\alpha_u\alpha_d\right] + h_{10}\left[\alpha_g(1 - \alpha_g) - \frac{3}{2}(\alpha_u^2 + \alpha_d^2)\right]\right], \quad (\text{A14})$$

$$\mathcal{A}_\perp(\underline{\alpha}) = 30\alpha_g^2(\alpha_u - \alpha_d)\left[h_{00} + h_{01}\alpha_g + \frac{1}{2}h_{10}(5\alpha_g - 3)\right], \quad (\text{A15})$$

where $C_n^m(\xi)$ are Gegenbauer polynomials.

The definitions and the specific forms of the η light-cone distribution amplitudes adopted in the text are similar to those of the pion. For more details see Ref. [31].